

Sia $y = a^x$

So che la f. inv. è $\log_a y = \log_a a^x \Rightarrow \log_a y = x$

Cioè $y = a^x \xrightarrow{-1} x = \log_a y$

La der. delle f. inv. è il reciproco delle derivate

Cioè $(a^x)' = \frac{1}{(\log_a y)'} = \frac{1}{\frac{1}{y} \log_e a} = \frac{y}{\log_e a} = y \log_e a = a^x \log_e a$

$(\log y)' = \frac{1}{y} \cdot (\log_e e) = 1$
 $\log_a y' = \frac{1}{y} \log_e a$

$\log_e a = \frac{1}{\log_a e}$

$a^x \log_e a$

$\Rightarrow \boxed{(a^x)' = a^x \log_e a}$

$$x^{5\cos x}$$

$$y = x^x \Rightarrow \log(y) = \log(x^x) = x \log x$$

$$(\log y)' = (x \log x)' \Rightarrow \frac{1}{y} \cdot y' = \log x + \frac{x}{x} = \log x + 1$$

$$\Rightarrow y' = (\log x + 1)y \Rightarrow y' = (\log x + 1)x^x$$

$$y = x^{5\cos x} \quad \log y = \log x^{5\cos x} \Rightarrow \log y = 5\cos x \cdot \log x$$

$$(\log y)' = (5\cos x \cdot \log x)'$$

$$\frac{1}{y} \cdot y' = -5\sin x \cdot \log x + \frac{5\cos x}{x}$$

$$\Rightarrow y' = \left(-5\sin x \log x + \frac{5\cos x}{x} \right) x^{5\cos x}$$

$$y = x^{5 \cos x}$$

(I method)

$$\log y = \log (x^{5 \cos x})$$

$$e^{\log y} = e^{\log (x^{5 \cos x})}$$

$$y = e^{\log (x^{5 \cos x})}$$

$$y = e^{5 \cos x \cdot \log x}$$

$$y' = e^{5 \cos x \cdot \log x} \cdot (5 \cos x \cdot \log x)'$$

$$y' = e^{5 \cos x \cdot \log x} \cdot \left(-5 \sin x \log x + \frac{5 \cos x}{x} \right)$$

$$y = x^{5 \cos x}$$

$$\log(y) = \log(x^{5 \cos x})$$

$$\log(y) = 5 \cos x \cdot \log x$$

$$(\log y)' = (5 \cos x \cdot \log x)'$$

$$\frac{1}{y} \cdot y' = -5 \sin x \log x + \frac{5 \cos x}{x}$$

$$y' = \left(-5 \sin x \cdot \log x + \frac{5 \cos x}{x} \right) y$$

$$y' = \left(-5 \sin x \cdot \log x + \frac{5 \cos x}{x} \right) x^{5 \cos x}$$

$$y = x^{4 \cos x}$$

$$\log(y) = \log(x^{4 \cos x})$$

$$\log y = 4 \cos x \log x$$

$$(\log y)' = (4 \cos x \log x)'$$

$$\frac{1}{y} \cdot y' = -4 \sin x \log x + \frac{4 \cos x}{x}$$

$$y' = \left(-4 \sin x \log x + \frac{4 \cos x}{x} \right) y$$

$$y' = \left(-4 \sin x \log x + \frac{4 \cos x}{x} \right) x^{4 \cos x}$$

$$y = x^{4 \cos x}$$

$$\log y = \log x^{4 \cos x}$$

$$e^{\log y} = e^{\log x^{4 \cos x}}$$

$$y = e^{\log x^{4 \cos x}} = e^{4 \cos x \cdot \log x}$$

$$y' = e^{4 \cos x \log x} (4 \cos x \cdot \log x)'$$

$$y' = e^{4 \cos x \log x} \left(-4 \sin x \log x + \frac{4 \cos x}{x} \right)$$

$$y = x^{\frac{3}{4 \log^2 x}}$$

$$\log y = \log x^{\frac{3}{4 \log^2 x}} = \frac{3}{4 \log^2 x} \cdot \log x$$

$$(\log y)' = \left(\frac{3}{4 \log^2 x} \cdot \log x \right)' = \left(\frac{3}{4 \log x} \right)'$$

$$\frac{1}{y} \cdot y' = \frac{3}{4} \cdot \frac{-\frac{1}{x}}{\log^2 x} = -\frac{3}{4x \log^2 x}$$

$$y' = -\frac{3}{4x \log^2 x} \cdot x^{\frac{3}{4 \log^2 x}} = -\frac{3x^{\frac{3}{4 \log^2 x}}}{4x \log^2 x}$$

$$y = x^{\frac{3}{4 \log^2 x}} = e^{\log x^{\frac{3}{4 \log^2 x}}}$$

$$\log_e x^{\frac{3}{4 \log^2 x}} = k \text{ t.c. } e^k = x^{\frac{3}{4 \log^2 x}}$$

$$\log_e (y) = \log_e \left(x^{\frac{3}{4 \log^2 x}} \right)$$

$$y = e^{\log \left(x^{\frac{3}{4 \log^2 x}} \right)} = e^{\frac{3}{4 \log^2 x} \cdot \log x} = e^{\frac{3}{4 \log x}}$$

$$y' = e^{\frac{3}{4 \log x}} \cdot \left(\frac{3}{4 \log x} \right)' = e^{\frac{3}{4 \log x}} \cdot \frac{3}{4} \cdot \frac{-\frac{1}{x}}{\log^2 x} =$$

$$= (-) e^{\frac{3}{4 \log x}} \cdot \frac{3}{4x \log^2 x} =$$