

$$\int \frac{1}{1+e^x} dx \quad \text{frattini parti}$$

$$\int \frac{\log 2x}{x \log 4x} dx \quad \text{metodo fpl}$$

$$\int \sqrt{2^x-1} dx \quad \text{sostituzione}$$

$$\int x \log \sqrt{1-x} \quad \text{per parti}$$

$$\int \frac{\arcsin x}{\sqrt{1-x^2}} \quad \text{sostituzione}$$

$$\int \frac{\arcsin x}{\sqrt{1-x}} \quad \text{part. + per parti}$$

$$\int \frac{\log^3 x}{x^2} dx \quad \text{per parti. più volte}$$

$$\int x^2 \log(1+x) dx \quad \text{per parti divisione polinomi}$$

$$\int \frac{2^x}{1-4^x} dx \quad \text{sostituzione}$$

$$\int \frac{1+e^x}{x+e^x} dx \quad \text{sostituzione immediata}$$

$$\int \frac{\sqrt{2-x}}{\sqrt{2+x}} dx \quad \text{sostituzione}$$

$$\int \frac{\sqrt{3-x}}{\sqrt{3+x}} dx \quad \text{sostituzione}$$

$$\int \frac{x^2 \arctan x}{1+x^2} dx \quad \text{sostituzione per parti}$$

$$\int \frac{e^{2x} + \sin 2x}{e^{2x} - \cos 2x} dx \quad \text{sostituzione}$$

$$\int \sqrt{2^x - 1} \, dx$$

$$\sqrt{2^x - 1} = t$$

$$2^x - 1 = t^2$$

$$2^x = t^2 + 1$$

$$\log_2 2^x = \log_2 |t^2 + 1| \Rightarrow x = \log_2 |t^2 + 1|$$

$$\Rightarrow dx = \frac{1}{t^2 + 1} \cdot 2t \cdot \frac{1}{\ln 2} = \frac{2}{\ln 2} \frac{t}{t^2 + 1}$$

$$= \int t \cdot \frac{2}{\ln 2} \frac{t}{t^2 + 1} = \frac{2}{\ln 2} \int \frac{t^2}{t^2 + 1} = \left\| \begin{array}{r} t^2 + 0t + 0 \\ t^2 \quad +1 \\ \hline -1 \end{array} \right\| \frac{t^2 + 1}{1}$$

$$= \frac{2}{\ln 2} \int 1 - \frac{1}{t^2 + 1} dt = \frac{2}{\ln 2} \int dt - \frac{2}{\ln 2} \int \frac{1}{t^2 + 1} = \frac{2}{\ln 2} t - \frac{2}{\ln 2} \arctan t$$

$$= \frac{2}{\ln 2} \sqrt{2^x - 1} - \frac{2}{\ln 2} \arctan \sqrt{2^x - 1} + C$$

VERIFICA:

$$\left(\frac{2}{\ln 2} \sqrt{2^x - 1} - \frac{2}{\ln 2} \arctan \sqrt{2^x - 1} \right)' = \frac{2}{\ln 2} \cdot \frac{1}{2\sqrt{2^x - 1}} \cdot 2^x \cdot \ln 2 -$$

$$- \frac{2}{\ln 2} \cdot \frac{1}{2^x - 1 + 1} \cdot \frac{1}{2\sqrt{2^x - 1}} \cdot 2^x \ln 2 = \frac{2^x}{\sqrt{2^x - 1}} - \frac{1}{\sqrt{2^x - 1}} =$$

$$= \frac{2^x - 1}{\sqrt{2^x - 1}} \cdot \left(\frac{\sqrt{2^x - 1}}{\sqrt{2^x - 1}} \right) = \frac{(2^x - 1) \sqrt{2^x - 1}}{2^x - 1} = \sqrt{2^x - 1} \quad \checkmark$$

$$\int \frac{1}{1+e^x} dx$$

$$\begin{aligned} e^x &= t \\ \log e^x &= \log t \\ x &= \log t \\ dx &= \frac{1}{t} dt \end{aligned}$$

$$\Rightarrow \int \frac{1}{1+t} \cdot \frac{1}{t} dt = \int \frac{1}{t(1+t)} dt$$

METODO DELLA DECOMPOSIZIONE IN FRAZIONI PARZIALI

$$\begin{aligned} \frac{1}{t(1+t)} &= \frac{A}{t} + \frac{B}{1+t} = \frac{A(1+t) + Bt}{t(1+t)} = \frac{A + At + Bt}{t(1+t)} \\ &= \frac{(A+B)t + A}{t(1+t)} \end{aligned}$$

$$\begin{cases} A+B=0 \\ A=1 \end{cases} \quad \text{per } 1+B=0 \Rightarrow B=-1$$

Allora $\frac{1}{t(1+t)} = \frac{1}{t} - \frac{1}{t+1}$

$$\begin{aligned} \Rightarrow \int \frac{1}{t(t+1)} dt &= \int \frac{1}{t} dt - \int \frac{1}{t+1} dt = \log t - \log |t+1| = \\ &= \log e^x - \log(e^x + 1) = x - \log(e^x + 1) + C \end{aligned}$$

VERIFICA: $\left[x - \log(e^x + 1) \right]' = 1 - \frac{1}{e^x + 1} \cdot e^x = \frac{e^x + 1 - e^x}{e^x + 1} = \frac{1}{e^x + 1} \quad \checkmark$

$$\int \frac{\arcsin^2 x}{\sqrt{1-x^2}} dx =$$

$$\arcsin x = t$$

$$\frac{1}{\sqrt{1-x^2}} dx = dt$$

$$= \int t^2 dt = \frac{t^3}{3} =$$

$$= \frac{\arcsin^3 x}{3} + C$$

$$\text{VERIFICA: } \left(\frac{\arcsin^3 x}{3} \right)' = \frac{1}{3} \cdot 3 \cdot \frac{\arcsin^2 x}{\sqrt{1-x^2}} = \frac{\arcsin^2 x}{\sqrt{1-x^2}} \quad \checkmark$$

$$\int \frac{\arcsin \sqrt{x}}{\sqrt{1-x}} dx$$

$$\arcsin \sqrt{x} = t \Rightarrow \sqrt{x} = \sin t$$

$$\frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}} dx = dt \Rightarrow \frac{dx}{\sqrt{1-x}} = 2 \sin t dt \quad (*)$$

$$\int \frac{t \cdot 2 \sin t}{\cancel{2\sqrt{x}}} dt = 2 \int t \sin t = 2 \left(-t \cos t + \int \cos t dt \right) =$$

$$= -2t \cos t + 2 \sin t = -2t \sqrt{1-x} + 2 \sin t =$$

$$= -2 \arcsin \sqrt{x} \sqrt{1-(\sin(\arcsin \sqrt{x}))^2} + 2 \sin(\arcsin \sqrt{x}) =$$

$$= -2 \arcsin \sqrt{x} \sqrt{1-(\sqrt{x})^2} + 2 \sin 2\sqrt{x} =$$

$$= -2 \arcsin \sqrt{x} \cdot \sqrt{1-x} + 2\sqrt{x} + C$$

VERIFICA:

$$(-2 \arcsin \sqrt{x} \cdot \sqrt{1-x} + 2\sqrt{x})' = -2 \left(\frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}} \cdot \sqrt{1-x} + \frac{\arcsin \sqrt{x}}{2\sqrt{1-x}} \cdot (-1) \right) + 2 \cdot \frac{1}{2\sqrt{x}} =$$

$$= -\frac{1}{\sqrt{x}} + \frac{\arcsin \sqrt{x}}{\sqrt{1-x}} + \frac{1}{\sqrt{x}} = \frac{\arcsin \sqrt{x}}{\sqrt{1-x}} \quad \checkmark$$

(*) Sol. alternative: $\arcsin \sqrt{x} = t \Rightarrow \sqrt{x} = \sin t \Rightarrow x = \sin^2 t$
 $dx = 2 \sin t \cos t dt$

$$\Rightarrow \int \frac{t}{\sqrt{1-\sin^2 t}} \cdot 2 \sin t \cos t dt = 2 \int \frac{t \cdot \sin t \cos t}{\cos t} dt = 2 \int t \cdot \sin t dt$$

$$\int \frac{\log^3 x}{x^2} dx \quad \text{peu până pînă vîrte}$$

NB $\int x^a dx = \frac{x^{a+1}}{a+1} = -\frac{1}{x}$

$$\int f'(x)g(x) dx = f(x)g(x) - \int f(x)g'(x) dx$$

$$\begin{cases} f'(x) = \frac{1}{x^2} \Rightarrow f(x) = -\frac{1}{x} \\ g(x) = \log^3 x \Rightarrow g'(x) = \frac{3\log^2 x}{x} \end{cases} \quad \left\| \right. = -\frac{1}{x} \log^3 x - \int -\frac{1}{x} \cdot \frac{3\log^2 x}{x} dx$$

$$= -\frac{1}{x} \log^3 x + 3 \int \frac{\log^2 x}{x^2} dx = -\frac{\log^3 x}{x} + 3 \left(-\frac{1}{x} \log^2 x - \int -\frac{1}{x} \cdot \frac{2\log x}{x} dx \right)$$

$$\begin{cases} f'(x) = \frac{1}{x^2} \Rightarrow f(x) = -\frac{1}{x} \\ g(x) = \log^2 x \Rightarrow g'(x) = \frac{2\log x}{x} \end{cases}$$

$$= -\frac{\log^3 x}{x} - \frac{3\log^2 x}{x} + 6 \int \frac{\log x}{x^2} dx = \begin{cases} f'(x) = \frac{1}{x^2} \Rightarrow f(x) = -\frac{1}{x} \\ g(x) = \log x \Rightarrow g'(x) = \frac{1}{x} \end{cases}$$

$$= -\frac{\log^3 x}{x} - \frac{3\log^2 x}{x} + 6 \left(-\frac{1}{x} \log x - \int -\frac{1}{x} \cdot \frac{1}{x} dx \right) =$$

$$= -\frac{\log^3 x}{x} - \frac{3\log^2 x}{x} - \frac{6\log x}{x} + 6 \int \frac{dx}{x^2} =$$

$$= -\frac{\log^3 x}{x} - \frac{3\log^2 x}{x} - \frac{6\log x}{x} - \frac{6}{x} + C$$

$$\int x^2 \log(1+x) dx =$$

$$\int f'(x) g(x) = f(x) g(x) - \int f(x) g'(x)$$

$$f'(x) = x^2 \Rightarrow f(x) = \frac{x^3}{3}$$

$$g(x) = \log(1+x) \Rightarrow g'(x) = \frac{1}{1+x}$$

$$= \frac{x^3}{3} \log(1+x) - \int \frac{x^3}{3} \cdot \frac{1}{1+x} dx = \frac{x^3}{3} \log(1+x) - \frac{1}{3} \int \frac{x^3}{1+x} dx =$$

$$\begin{array}{r} x^3 + 0x^2 + 0x + 0 \\ -x^3 - x^2 \\ \hline // -x^2 \\ +x^2 + x \\ \hline // x \\ -x - 1 \\ \hline // -1 \end{array} \quad \frac{x^3}{x+1} = x^2 - x + 1 - \frac{1}{x+1}$$

$$= \frac{x^3}{3} \log(1+x) - \frac{1}{3} \int \left(x^2 - x + 1 - \frac{1}{x+1} \right) dx =$$

$$= \frac{x^3}{3} \log(1+x) - \frac{1}{3} \int x^2 dx + \frac{1}{3} \int x dx - \frac{1}{3} \int dx + \frac{1}{3} \int \frac{1}{x+1} dx$$

$$= \frac{x^3}{3} \log(1+x) - \frac{1}{3} \frac{x^3}{3} + \frac{1}{3} \frac{x^2}{2} - \frac{1}{3} x + \frac{1}{3} \log|x+1| + C$$

$$= \frac{x^3 \log(1+x)}{3} - \frac{x^3}{9} + \frac{x^2}{6} - \frac{x}{3} + \frac{\log|x+1|}{3}$$

$$\frac{1}{3} \left(3x^2 \log(1+x) + \frac{x^3}{1+x} \right) - \frac{3x^2}{9} + \frac{2x}{6} - \frac{1}{3} + \frac{1}{3(x+1)} =$$

$$= \frac{3x^2 \log(1+x)}{3} + \frac{x^3}{3(1+x)} - \frac{x^2}{3} + \frac{x}{3} - \frac{1}{3} + \frac{1}{3(x+1)} =$$

$$= \frac{[3x^2 \log(1+x)](1+x) + x^3 - x^2(x+1) + x(x+1) - (x+1) + 1}{3(1+x)} =$$

$$= \frac{3x^2 \log(1+x) + 3x^3 \log(1+x) + \cancel{x^3} - \cancel{x^3} - \cancel{x^2} + \cancel{x^2} + \cancel{x} - \cancel{x} - \cancel{1} + 1}{3(1+x)} =$$

$$= \frac{3x^2 \log(1+x) [1+x]}{3(1+x)} = x^2 \log(1+x) \quad \checkmark$$

$$\int \frac{2^x}{1-4^x} dx =$$

$$(4)^x = (2 \cdot 2)^x = 2^x \cdot 2^x$$

$$(2^x)' = 2^x \cdot \log 2$$

$$= \int \frac{2^x dx}{1 - (2^x)^2} =$$

$$2^x = t \Rightarrow 2^x dx = \frac{dt}{\log 2}$$

$$= \frac{1}{\log 2} \int \frac{dt}{1-t^2} = \frac{1}{\log 2} \cdot \frac{1}{2} \log \left| \frac{1+t}{1-t} \right| = \frac{1}{2 \log 2} \log \left| \frac{1+2^x}{1-2^x} \right|$$

VER.

$$\frac{1}{2 \log 2} \left(\frac{1-2^x}{1+2^x} \cdot \frac{2^x \log 2 (1-2^x) - (1+2^x) \cdot (-2^x \log 2)}{(1-2^x)^2} \right) =$$

$$= \frac{1}{2 \log 2} \cdot \frac{2^x \log 2 (1-2^x) + 2^x \log 2 (1+2^x)}{1 - (2^x)^2} = \frac{1}{2 \log 2} \cdot \frac{2^x \log 2 (1-2^x + 1+2^x)}{1-4^x} =$$

$$= \frac{2 \cdot 2^x}{2(1-4^x)} = \frac{2^x}{1-4^x} \quad \checkmark$$

$$\int \frac{1+e^x}{x+e^x} dx$$

$$x+e^x=t$$

$$(1+e^x)dx=dt$$

$$\int \frac{dt}{t} = \log |t| = \log |x+e^x| + C$$

verification

$$\left(\log |x+e^x| \right)' = \frac{1}{x+e^x} \cdot (1+e^x) = \frac{1+e^x}{x+e^x} \quad \checkmark$$

$$\int \frac{\sqrt{2-x}}{\sqrt{2+x}} dx \cdot \frac{\sqrt{2-x}}{\sqrt{2-x}} = \int \frac{2-x}{\sqrt{(2+x)(2-x)}} = \int \frac{2-x}{\sqrt{4-x^2}} dx$$

$$= 2 \int \frac{dx}{\sqrt{4-x^2}} - \int \frac{x dx}{\sqrt{4-x^2}} = \left\| \begin{array}{l} x^2 = t \\ 2x dx = \frac{dt}{2} \end{array} \right.$$

$$= 2 \arcsin \frac{x}{2} - \frac{1}{2} \int \frac{dt}{\sqrt{4-t}} = \left\| \begin{array}{l} \sqrt{4-t} = z \\ -\frac{1}{2\sqrt{4-t}} dt = dz \end{array} \right.$$

$$= 2 \arcsin \frac{x}{2} + \int dz =$$

$$= 2 \arcsin \frac{x}{2} + z = 2 \arcsin \frac{x}{2} + \sqrt{4-t} = 2 \arcsin \frac{x}{2} + \sqrt{4-x^2} + C$$

$$\int \frac{\sqrt{3-x}}{\sqrt{3+x}} dx \cdot \frac{\sqrt{3-x}}{\sqrt{3-x}} = \int \frac{3-x}{\sqrt{(3+x)(3-x)}} dx = \int \frac{3-x}{\sqrt{9-x^2}} dx =$$

$$\int \frac{3 dx}{\sqrt{9-x^2}} - \int \frac{x dx}{\sqrt{9-x^2}} = 3 \int \frac{dx}{\sqrt{9-x^2}} - \frac{1}{2} \int \frac{dt}{\sqrt{9-t}} \quad \left\| \begin{array}{l} x^2 = t \\ 2x dx = \frac{dt}{2} \end{array} \right\|$$

$$= 3 \int \frac{dx}{\sqrt{9-x^2}} + \int dz =$$

$$\left\| \begin{array}{l} \sqrt{9-t} = z \\ -\frac{1}{2\sqrt{9-t}} dt = dz \end{array} \right\|$$

$$= 3 \arcsin \frac{x}{3} + z = 3 \arcsin \frac{x}{3} + \sqrt{9-t} =$$

$$= 3 \arcsin \frac{x}{3} + \sqrt{9-x^2} + C$$

$$\int \frac{x^2 \arctan x}{1+x^2} dx =$$

$$\arctan x = t \Rightarrow x = \tan t$$

$$\frac{1}{1+x^2} dx = dt$$

$$= \int \tan^2 t \cdot t \, dt =$$

$$\text{So che } \int \tan^2 t \, dt = \int \sec^2 t - 1 \, dt = \int \sec^2 t \, dt - \int dt =$$

$$= \int \frac{1}{\cos^2 t} \, dt - \int dt = \tan t - t \Rightarrow (\tan t - t)' = \tan^2 t$$

per parti $\int f'(t) g(t) \, dt = f(t) g(t) - \int f(t) g'(t) \, dt$

con $f'(t) = \tan^2 t \Rightarrow f(t) = \tan t - t$

$g(t) = t \Rightarrow g'(t) = 1$

$$= (\tan t - t) t - \int \tan t - t \, dt = t \tan t - t^2 - \int \tan t \, dt + \int t \, dt =$$

$$= t \cdot \tan t - t^2 + \log |\sec t| + \frac{t^2}{2} = t \tan t - \frac{t^2}{2} + \log |\sec t| =$$

$$= \arctan x \cdot x - \frac{1}{2} \arctan^2 x + \log |\cos(\arctan x)| + C$$

$$\int \frac{e^{2x} + 2x e^{2x}}{e^{2x} - \cos 2x} dx =$$

$$e^{2x} - \cos 2x = t$$

$$(2e^{2x} + 2x e^{2x}) dx = dt$$

$$(e^{2x} + x e^{2x}) dx = \frac{dt}{2}$$

$$= \frac{1}{2} \int \frac{dt}{t} = \frac{1}{2} \log |t| = \frac{1}{2} \log |e^{2x} - \cos 2x|$$